Square and Cube Root Equations

Perfect Square: the product of two equal integers. Ex: $3 \cdot 3 = \underline{9}$
↑ perfect square

Cube (Perfect Cube): the product of three equal integers. Ex: $2 \cdot 2 \cdot 2 = \underline{8}$
↑ perfect cube

* Non-perfect squares and cubes are irrational.
* Squaring a number and taking its square root are inverse operations.

Ex1) $3 \cdot 3 = 9$, so the square root of $9 = 3$.

Ex2) $4^2 = 16$ $\quad\quad$ Ex3) $(-5)^2 = 25$

$\sqrt{16} = \pm 4$ $\quad\quad$ $\sqrt{25} = \pm 5$

* Cubing a number and taking the cube root $\sqrt[3]{\ }$ are inverse operations.

Ex1) $(-2)(-2)(-2) = -8$ $\quad\quad$ Ex2) $4 \cdot 4 \cdot 4 = 64$

$\sqrt[3]{-8} = (-2)$ $\quad\quad$ $\sqrt[3]{64} = 4$

Ex3) $5 \cdot 5 \cdot 5 = 125$, so the cube root of $125 = 5$.

Example 1) $4^2 = 16$

$\sqrt{16} = \pm 4$, because $-4 \cdot -4 = 16$

$4 \cdot 4 = 16$

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Example 2) $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \boxed{\frac{1}{3}}$

Example 3) $\left(\frac{1}{2}\right)^2 = \left(\frac{1^2}{2^2}\right) = \frac{1}{4}$ and $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \boxed{\frac{1}{2}}$

Example 4) solve: $x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

* There are two solutions because $5 \cdot 5$ and $-5 \cdot -5$ will both equal 25.

Example 5) Solve. $x^2 = 36$

$\sqrt{x^2} = \sqrt{36}$

$x = \pm 6$

* There are two solutions because $6 \cdot 6$ and $-6 \cdot -6$ will both equal 36.

Example 6) solve: $x^2 = \frac{4}{9}$

$\sqrt{x^2} = \sqrt{\frac{4}{9}}$

$x = \frac{\sqrt{4}}{\sqrt{9}} = \boxed{\frac{\pm 2}{3}}$

Example 7) $x^3 = 27$

$\sqrt[3]{x^3} = \sqrt[3]{27}$

$\boxed{x = 3}$

Example 8) $x^3 = \frac{1}{8}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{1}{8}}$

$x = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \boxed{\frac{1}{2}}$

Example 9) What is the side length of a square with an area of $49\ ft^2$?

7

7 { $\boxed{A = 49\ ft^2}$

$A = s^2$

$49 = 7^2$

$\sqrt{49} = 7$

The length of one side is 7ft.