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Square and Cube Root Equations

Perfect Square: the product of two equal integers. Ex: $3 \cdot 3 = 9$

$$\begin{array}{c} \underline{9} \\ \uparrow \\ \text{perfect square} \end{array}$$

Cube (Perfect Cube): the product of three equal integers. Ex: $2 \cdot 2 \cdot 2 = 8$

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perfect cube

* Non-perfect squares and cubes are irrational.

* Squaring a number and taking its square root are inverse operations.

Ex1) $3 \cdot 3 = 9$, so the square root of $9 = 3$.

Ex2) $4^2 = 16$

$$\sqrt{16} = \pm 4$$

Ex3) $(-5)^2 = 25$

$$\sqrt{25} = \pm 5$$

* Cubing a number and taking the cube root $\sqrt[3]{\quad}$ are inverse operations.

Ex1) $(-2)(-2)(-2) = -8$

$$\sqrt[3]{-8} = (-2)$$

Ex2) $4 \cdot 4 \cdot 4 = 64$

$$\sqrt[3]{64} = 4$$

Ex3) $5 \cdot 5 \cdot 5 = 125$, so the cube root of $125 = 5$.

Example 1) $4^2 = 16$

$$\sqrt{16} = \pm 4, \text{ because } -4 \cdot -4 = 16$$

$$4 \cdot 4 = 16$$

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Example 2) $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$

Example 3) $\left(\frac{1}{2}\right)^2 = \left(\frac{1^2}{2^2}\right) = \frac{1}{4}$ and $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$

Example 4) solve. $x^2 = 25$
 $\sqrt{x^2} = \sqrt{25}$
 $x = \pm 5$

* There are two solutions because $5 \cdot 5$ and $-5 \cdot -5$ will both equal 25.

Example 5) Solve. $x^2 = 36$
 $\sqrt{x^2} = \sqrt{36}$
 $x = \pm 6$

* There are two solutions because $6 \cdot 6$ and $-6 \cdot -6$ will both equal 36.

Example 6) solve. $x^2 = \frac{4}{9}$

$$\sqrt{x^2} = \sqrt{\frac{4}{9}}$$

$$x = \frac{\sqrt{4}}{\sqrt{9}} = \frac{\pm 2}{3}$$

Example 7) $x^3 = 27$

$$\sqrt[3]{x^3} = \sqrt[3]{27}$$

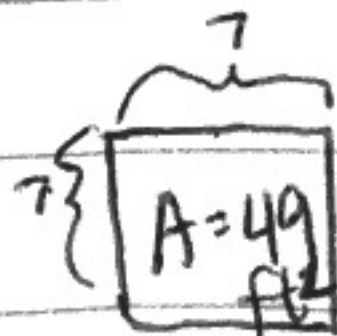
$$x = 3$$

Example 8) $x^3 = \frac{1}{8}$

$$\sqrt[3]{x^3} = \sqrt[3]{\frac{1}{8}}$$

$$x = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}$$

Example 9) What is the side length of a square with an area of 49 ft^2 ?



$$A = s^2$$
$$49 = 7^2$$

$\sqrt{49} = 7$
The length of one side is 7ft.