

## 6.4 Applications of Linear Systems (D1)

SWBAT translate a break-even word problem into a system of linear equations and solve.

### Finding a Break-Even Point

Step 1: READ the problem

Step 2: Underline or HIGHLIGHT the QUESTION

Step 3: Define the VARIABLES (they are found in the question)

Step 4: Reread the problem and WRITE THE EQUATION (one cost, one profit)

Step 5: Set the EQUATIONS EQUAL to each other and solve!

1. A fashion designer makes and sells hats. The material for each hat costs \$5.50. The hats sell for \$12.50 each. The designer spends \$1400 on advertising. How many hats must the designer sell to break even?

**Set Up:**

Let  $X = \underline{\text{\# OF HATS SOLD}}$

Let  $Y = \underline{\text{\# OF \$ PROFIT OR INCOME}}$

**Equations:**

$$5.5x + 1400 \quad (\text{COST})$$

$$12.5x \quad (\text{PROFIT})$$

**SOLVE:**

$$12.5x = 5.5x + 1400$$

$$\underline{-5.5x \quad -5.5x}$$

$$\frac{7x}{7} = \frac{1400}{7} \quad \boxed{X = 200}$$

200 HATS MUST BE SOLD

2. A puzzle expert wrote a new Sudoku puzzle book. His initial costs are \$864. Binding and packaging each book cost \$0.80. The price of the book is \$2. How many copies must be sold to break even?

**Set Up:**

Let  $X = \underline{\text{\# OF BOOKS SOLD}}$

Let  $Y = \underline{\text{\# OF \$ OF INCOME}}$

**Equations:**

$$y = \$.80x + 864$$

$$y = \$2x$$

**SOLVE:**

$$2x = .80x + 864$$

$$\underline{-.80x \quad -.80x}$$

$$\frac{1.20x}{1.20} = \frac{864}{1.20} \quad \boxed{X = 720}$$

720 COPIES MUST BE SOLD

3. A bicycle store costs \$2400 per month to operate. The store pays an average of \$60 per bike. The average selling price of each bicycle is \$120. How many bicycles must the store sell each month to break even?

**Set Up:**

Let  $X = \underline{\text{\# OF BICYCLES SOLD}}$

Let  $Y = \underline{\text{\# OF \$ OF INCOME}}$

**Equations:**

$$y = \$60x + 2400$$

$$y = 120x$$

**SOLVE:**

$$120x = 60x + 2400$$

$$\underline{-60x \quad -60x}$$

$$\frac{60x}{60} = \frac{2400}{60} \quad \boxed{X = 40}$$

40 BICYCLES MUST BE SOLD TO BREAK EVEN

4. Producing a musical cost \$88,000 plus \$5900 per performance. One sold-out performance earns \$7500 in revenue. If every performance sells out, how many performances are needed to break even?

**Set Up:**

Let  $X = \underline{\text{\# OF PERFORMANCES}}$

Let  $Y = \underline{\text{\$ OF REVENUE}}$

**Equations:**

$$y = 5900x + 88000$$

$$y = 7500x$$

**SOLVE:**

$$7500x = 5900x + 88000$$

$$\underline{-5900x \quad -5900x}$$

$$\frac{1600x}{1600} = \frac{88000}{1600} \quad \boxed{X = 55}$$

55 PERFORMANCES

5. A carpenter makes and sells rocking chairs. The material for each chair costs \$22.50. The chairs sell for \$75 each. If the carpenter spends \$420 on advertising, how many chairs must she sell to break even?

**Set Up:**

Let  $X = \underline{\text{\# OF CHAIRS SOLD}}$

Let  $Y = \underline{\text{\$ OF REVENUE}}$

**SOLVE:**

$$\begin{array}{r} 75x = 22.5x + 420 \\ -22.5x \quad -22.5x \\ \hline 52.5x = 420 \\ \underline{52.5} \quad \underline{52.5} \\ x = 8 \end{array}$$

**Equations:**

$$\begin{array}{l} y = 22.5x + 420 \\ y = 75x \end{array}$$

8 CHAIRS MUST BE SOLD

### Solving Word Problems with Two Variables

**Step 1:** Read the problem

**Step 2:** Underline or highlight the question

**Step 3:** Define the variables (they are found in the question)

**Step 4:** Reread the problem and write the equations

**Step 5:** Solve!

6. The sum of two numbers is 73. When the smaller number is subtracted from twice the greater number, the result is 50. Find the two numbers.

**Set Up:**

Let  $X = \underline{\text{larger \#}}$

Let  $Y = \underline{\text{smaller \#}}$

**SOLVE:**

$$\begin{array}{r} 41 + y = 73 \\ -41 \quad \quad -41 \\ \hline y = 32 \end{array}$$

**Equations:**

$$\begin{array}{l} x + y = 73 \\ 2x - y = 50 \\ \hline 3x = 123 \\ \underline{3} \quad \quad \quad \underline{3} \\ x = 41 \end{array}$$

7. The length of a rectangle is 5 cm less than three times its width. If the perimeter is 70 cm, find the area of the rectangle.

**Set Up:**

Let  $X = \underline{\text{length}}$

Let  $Y = \underline{\text{width}}$

**SOLVE:**

$$\begin{array}{l} 2(3y - 5) + 2y = 70 \\ 6y - 10 + 2y = 70 \\ 8y - 10 + 10 = 70 + 10 \\ 8y = 80 \\ \underline{8} \quad \quad \underline{8} \\ y = 10 \end{array}$$

$$\begin{array}{l} x = 3y - 5 \\ x = 3(10) - 5 \\ = 30 - 5 \\ \boxed{x = 25} \end{array}$$

**Equations:**

$$\begin{array}{l} x = 3y - 5 \\ 2x + 2y = 70 \end{array}$$

$$\begin{array}{l} \text{Area} = l \cdot w = \\ 25 \cdot 10 \\ = 250 \end{array}$$

8. John has 15 coins, all dimes and quarters, worth \$2.55. How many dimes and how many quarters does John have?

**Set Up:**

Let  $X = \underline{\text{\# of dimes}}$

Let  $Y = \underline{\text{\# of quarters}}$

**SOLVE:**

$$\begin{array}{l} -10(x + y = 15) \\ 100(.10x + .25y = 2.55) \\ \boxed{8 \text{ Dimes } 7 \text{ quarters}} \end{array}$$

**Equations:**

$$\begin{array}{l} x + y = 15 \\ .10x + .25y = 2.55 \\ \hline -10x - 10y = -150 \\ 10x + 25y = 255 \\ \hline 15y = 105 \quad \boxed{y = 7} \\ x + 7 = 15 \\ -7 \quad -7 \\ \hline \boxed{x = 8} \end{array}$$

9. Tickets for the senior play cost \$4 for adults and \$2 for students. This year there were 600 tickets sold, and the class made \$1900. How many of each type of ticket was sold?

**Set Up:**

Let  $X =$  # of adult tickets  
 Let  $Y =$  # of student tickets

**Equations:**

$$\begin{aligned} X + Y &= 600 \\ 4X + 2Y &= 1900 \end{aligned}$$

**SOLVE:**

$$\begin{aligned} -4(X + Y = 600) &= -4X - 4Y = -2400 \\ 4X + 2Y &= 1900 \\ \hline -2Y &= -500 \\ -2 & \quad -2 \\ \hline Y &= 250 \end{aligned}$$

$$\begin{aligned} X + 250 &= 600 \\ -250 & \quad -250 \\ \hline X &= 350 \end{aligned}$$

10. Kathleen invested \$5000, some at 6% and the rest at 5%. Her annual income from the investments is \$280. How much is invested at 5%?

**Set Up:**

Let  $X =$  # invested @ 6%  
 Let  $Y =$  # invested @ 5%

**Equations:**

$$\begin{aligned} X + Y &= 5000 \\ .06X + .05Y &= 280 \end{aligned}$$

**SOLVE:**

$$\begin{aligned} 100(.06X + .05Y = 280) &= 6X + 5Y = 28000 \\ -6(X + Y = 5000) &= -6X - 6Y = -30000 \\ \hline -Y &= -2000 \\ Y &= 2000 \end{aligned}$$

11. A baseball manager bought 4 bats and 9 balls for \$168.75. On another day, he bought 3 bats and 1 dozen balls for \$172.50. How much did he pay for each bat and each ball?

**Set Up:**

Let  $X =$  # of bats  
 Let  $Y =$  # of balls

**Equations:**

$$\begin{aligned} 4X + 9Y &= 168.75 \\ 3X + 12Y &= 172.50 \end{aligned}$$

**SOLVE:**

$$\begin{aligned} 3(4X + 9Y = 168.75) &= 12X + 27Y = 506.25 \\ -4(3X + 12Y = 172.50) &= -12X - 48Y = -690.00 \\ \hline -21Y &= -183.75 \\ Y &= 8.75 \end{aligned}$$

$$\begin{aligned} 4X + 9(8.75) &= 168.75 \\ 4X + 78.75 &= 168.75 \\ 4X &= 90 \\ X &= 22.5 \end{aligned}$$

12. CHALLENGE: You want to sell 1 lb jars of mixed peanuts and cashews for \$5. You pay \$3 per pound for peanuts and \$6 per pound for cashews. You plan to combine 4 parts peanuts and 1 part cashews to make your mix. You have spent \$70 on materials to get started. How many jars must you sell to break even?

**Set Up:**

Let \_\_\_\_\_ = \_\_\_\_\_  
 Let \_\_\_\_\_ = \_\_\_\_\_

**Equations:**

\_\_\_\_\_

**SOLVE:**

Solve each system. Explain why you chose the method you used.

14.  $4x + 5y = 3$   
 $3x - 2y = 8$

15.  $2x + 7y = -20$   
 $y = 3x + 7$

16.  $5x + 2y = 17$   
 $x - 2y = 8$

9. Tickets for the senior play cost \$4 for adults and \$2 for students. This year there were 600 tickets sold, and the class made \$1900. How many of each type of ticket was sold?

**Set Up:**

Let  $X =$  # of adult tickets  
 Let  $Y =$  # of student tickets

**Equations:**

$$\begin{aligned} X + Y &= 600 \\ 4X + 2Y &= 1900 \end{aligned}$$

SOLVE:  $4(x + y = 600) = -4x - 4y = -2400$   
 $4x + 2y = 1900$   

$$\begin{aligned} -4x - 4y &= -2400 \\ 4x + 2y &= 1900 \\ \hline -2y &= -500 \\ -2 & \quad -2 \\ \hline Y &= 250 \end{aligned}$$

$X + 250 = 600$   
 $-250 \quad -250$   
 $X = 350$

10. Kathleen invested \$5000, some at 6% and the rest at 5%. Her annual income from the investments is \$280. How much is invested at 5%?

**Set Up:**

Let  $X =$  \$ invested @ 6%  
 Let  $Y =$  \$ invested @ 5%

**Equations:**

$$\begin{aligned} X + Y &= 5000 \\ .06X + .05Y &= 280 \end{aligned}$$

SOLVE:  $100(.06x + .05y = 280)$   
 $6x + 5y = 28000$   
 $-6(x + y = 5000)$   

$$\begin{aligned} 6x + 5y &= 28000 \\ -6x - 6y &= -30000 \\ \hline -y &= -2000 \\ Y &= 2000 \end{aligned}$$

11. A baseball manager bought 4 bats and 9 balls for \$168.75. On another day, he bought 3 bats and 1 dozen balls for \$172.50. How much did he pay for each bat and each ball?

**Set Up:**

Let  $X =$  # of bats  
 Let  $Y =$  # of balls

**Equations:**

$$\begin{aligned} 4X + 9Y &= 168.75 \\ 3X + 12Y &= 172.50 \end{aligned}$$

SOLVE:  $3(4x + 9y = 168.75) = 12x + 27y = 506.25$   
 $-4(3x + 12y = 172.50) = -12x - 48y = -690.00$   

$$\begin{aligned} 12x + 27y &= 506.25 \\ -12x - 48y &= -690.00 \\ \hline -21y &= -183.75 \\ Y &= 8.75 \end{aligned}$$

$4x + 9(8.75) = 168.75$   
 $4x + 78.75 = 168.75$   
 $4x = 90$   
 $X = 22.5$

12. CHALLENGE: You want to sell 1 lb jars of mixed peanuts and cashews for \$5. You pay \$3 per pound for peanuts and \$6 per pound for cashews. You plan to combine 4 parts peanuts and 1 part cashews to make your mix. You have spent \$70 on materials to get started. How many jars must you sell to break even?

**Set Up:**

Let \_\_\_\_\_ = \_\_\_\_\_  
 Let \_\_\_\_\_ = \_\_\_\_\_

**Equations:**

\_\_\_\_\_

SOLVE:

Solve each system. Explain why you chose the method you used.

14.  $4x + 5y = 3$   
 $3x - 2y = 8$

15.  $2x + 7y = -20$   
 $y = 3x + 7$

16.  $5x + 2y = 17$   
 $x - 2y = 8$

14

$$\begin{aligned} 2(4x + 5y) &= 3 \\ 5(3x - 2y) &= 8 \end{aligned}$$

$$\begin{aligned} 8x + 10y &= 6 \\ 15x - 10y &= 40 \\ \hline 23x &= 46 \\ \frac{23x}{23} &= \frac{46}{23} \end{aligned}$$

ELIMINATION

$$\boxed{x = 2}$$

$$\begin{aligned} 8x + 10y &= 6 \\ 8(2) + 10y &= 6 \\ 16 + 10y &= 6 \\ -16 & \quad -16 \\ \hline 10y &= -10 \end{aligned}$$

$$\boxed{y = -1}$$

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$$\begin{aligned} 2x + 7y &= -20 \\ y &= 3x + 7 \\ y &= 3(-3) + 7 \\ &= -9 + 7 \\ \boxed{y} &= \boxed{-2} \end{aligned}$$

$$\begin{aligned} 2x + 7(3x + 7) &= -20 \\ 2x + 21x + 49 &= -20 \\ 23x - 49 &= -49 \\ \hline 23x &= -69 \\ \frac{23x}{23} &= \frac{-69}{23} \\ \boxed{x} &= \boxed{-3} \end{aligned}$$

Substitution

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$$\begin{aligned} 5x + 2y &= 17 \\ x - 2y &= 8 \\ \hline 6x &= 25 \\ \frac{6x}{6} &= \frac{25}{6} \\ \boxed{x} &= \frac{25}{6} \end{aligned}$$

$$\begin{aligned} x - 2y &= 8 \\ \frac{25}{6} - 2y &= 8 \\ -\frac{25}{6} &= -\frac{25}{6} \\ \hline -2y &= \frac{23}{6} \\ \frac{-2y}{2} &= \frac{23}{6} \div 2 \\ \boxed{y} &= \frac{-23}{12} \end{aligned}$$

ELIMINATION