

# Arithmetic Sequences

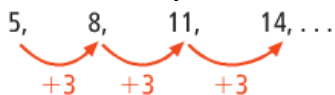
SWBAT identify and extend patterns in sequences and represent arithmetic sequences using function notation.

A **sequence** is an ordered list of numbers that often form a pattern. Each number in the list is called a **term of a sequence**.

**Essential Understanding:** When you can identify a pattern in a sequence, you can use it to extend the sequence. You can model some sequences with a function rule that you can use to find any term or the sequence.

**Extending Sequences:** Look how each term of the sequence is related to the previous term. Your goal is to identify a single rule that you can apply to every term to produce the next term.

### Example 1:



A pattern is "add 3 to the previous term." So the next two terms are  $14 + 3 = 17$  and  $17 + 3 = 20$ .

### Example 2:



A pattern is "multiply the previous term by 2." So the next two terms are  $2(20) = 40$  and  $2(40) = 80$ .

Describe a pattern in each sequence. What are the next two terms of each sequence:

a. 5, 11, 17, 23, ...

b. 400, 200, 100, 50, ...

c. 2, -4, 8, -16, ...

d. -15, -11, -7, -3

Practice: pg. 277 # 9-17

In an **arithmetic sequence**, the difference between consecutive terms is constant. This difference is called the **common difference**.

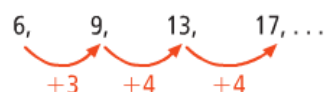
**Identifying an Arithmetic Sequence.** The difference between every pair of consecutive terms must be the same.

### Example 1:



The sequence has a common difference of 5, so it is arithmetic.

### Example 2:



The sequence does not have a common difference, so it is not arithmetic.

Tell whether the sequence is arithmetic. If it is, what is the common difference?

a. 8, 15, 22, 30, ...

b. 7, 9, 11, 13, ...

c. 10, 4, -2, -8, ...

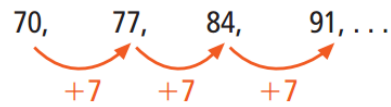
d. 2, -2, 2, -2, ...

Practice: pg. 277 18-29

A sequence is a function whose domain is the natural numbers, and whose outputs are the terms of the sequence. You can write a sequence using a recursive formula.

A **recursive formula** is a function rule that relates each term of a sequence after the first to the ones before it.

**Write a recursive formula for the arithmetic sequence below. What is the value of the 8th term?**



**Step 1:**

**Step 2:**

Write a recursive formula for each arithmetic sequence. What is the 9<sup>th</sup> term of each sequence?

a. 3, 9, 15, 21, ...

b. 23, 35, 47, 59, .....

c. 7.3, 7.8, 8.3, 8.8

d. 97, 88, 79, 70,.....

An **explicit formula** is a function rule that relates each term of a sequence to the term number.

You can find any term of an arithmetic sequence if you know the first term and the common difference.

The  $n$ th term of an arithmetic sequence with first term  $A(1)$  and common difference  $d$  is given by

$$A(n) = A(1) + (n - 1)d$$

$n$ th term    first term    term number    common difference



## Writing an Explicit Formula

**Online Auction:** An online auction works as shown below.

Write an explicit formula to represent the bids as an arithmetic sequence. What is the twelfth bid?

Make a table of the bids. Identify the first term and common difference.

Substitute  $A(1) = 200$  and  $d = 10$  into the formula  $A(n) = A(1) + (n - 1)d$ . The explicit formula  $A(n) = 200 + (n-1)10$  represents the arithmetic sequence of the auction bids. To find the twelfth bid, evaluate  $A(n)$  for  $n = 12$ .  $A(12) = 200 + (12 - 1)10 = 310$ . The twelfth bid is \$310.

**Practice:** A subway pass has a starting value of \$100. After one ride, the value of the pass is \$98.25. After two rides, its value is \$96.50. After three rides, its value is \$94.75. Write an explicit formula to represent the remaining value on the card as an arithmetic sequence. What is the value of the pass after 15 rides? How many rides can be taken with the \$100 pass?

**Writing an Explicit Formula from a Recursive Formula:** An arithmetic sequence is represented by the recursive formula  $A(n) = A(n-1) + 12$ . If the first term of the sequence is 19, write the explicit formula.

For each recursive formula, find an explicit formula that represents the same sequence.

a.  $A(n) = A(n - 1) + 2$ ;  $A(1) = 21$

b.  $A(n) = A(n - 1) + 7$ ;  $A(1) = 2$

**Writing a Recursive Formula From an Explicit Formula:** An arithmetic sequence is represented by the explicit formula  $A(n) = 32 + (n - 1)(22)$ . What is the recursive formula:

For each explicit formula, find a recursive formula that represents the same sequence.

a.  $A(n) = 76 + (n - 1)(10)$

b.  $A(n) = 1 + (n - 1)(3)$