

Key Concept Geometric Sequence

A geometric sequence with a *starting value* a and a *common ratio* r is a sequence of the form a, ar, ar^2, ar^3, \dots

A *recursive definition* for the sequence has two parts:

$$a_1 = a \quad \text{Initial condition}$$

$$a_n = a_{n-1} \cdot r, \text{ for } n \geq 2 \quad \text{Recursive formula}$$

An *explicit definition* for this sequence is a single formula:

$$a_n = a_1 \cdot r^{n-1}, \text{ for } n \geq 1$$

Every geometric sequence has a starting value and a common ratio. The starting value and common ratio define a unique geometric sequence.

1 Geometric Sequences

SWBAT write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Essential Understanding: In a geometric sequence, the ratio of any term to its preceding term is a constant value.

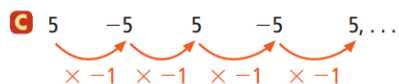
The recursive formula is useful for finding the next term in the sequence.

The explicit formula is more convenient when finding the n th term

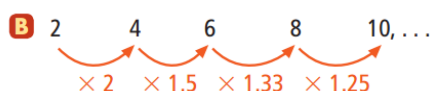
Identifying an Geometric Sequence. The common ratio between every pair of consecutive terms must be the same.

Example 1:

Example 3:



Example 2:



There is a common ratio, $r = 10$. So, the sequence is geometric.

There is no common ratio. So, the sequence is not geometric. There is a common ratio, $r = -1$. So, the sequence is geometric.

Tell whether the sequence is geometric. If the sequence is not geometric, is it arithmetic?

- a. 3, 6, 12, 24, 48, ... b. 3, 6, 9, 12, 15... c. $1/3, 1/9, 1/27, 1/81, \dots$ d. 4, 7, 11, 16, 22, ...

Find the recursive and explicit formulas for the sequence 7, 21, 63, 189, ...

The starting value a_1 is 7. The common ratio r is $\frac{21}{7} = 3$.

$$a_1 = a; a_n = a_{n-1} \cdot r$$

Use the formula.

$$a_n = a_1 \cdot r^{n-1}$$

$$a_1 = 7; a_n = a_{n-1} \cdot r$$

Substitute the starting value for a_1 .

$$a_n = 7 \cdot r^{n-1}$$

$$a_1 = 7; a_n = a_{n-1} \cdot 3$$

Substitute the common ratio for r .

$$a_n = 7 \cdot 3^{n-1}$$

The recursive formula is

$$a_1 = 7; a_n = a_{n-1} \cdot 3.$$

The explicit formula is

$$a_n = 7 \cdot 3^{n-1}.$$

Practice: Find the 8th term of each sequence

a. 14, 84, 504, 3024

b. 648, 324, 162, 81

Writing Geometric Sequences as Functions: A geometric sequence has an initial value of 6 and a common ratio of 2. Write a function to represent the sequence. Graph the function.

Practice: A geometric sequence has an initial value of 2 and a common ratio of 3. Write a function to represent the sequence. Graph the function.

Homework:

- Determine if each sequence is a geometric sequence. If it is, find the common ratio and write the explicit formula. Solve for the 9th term.
 - 5, 10, 20, 40,
 - 20, 15, 10, 5,
 - 3, -9, 27, -81
- Identify each sequence as arithmetic, geometric or neither.
 - 1.5, 4.5, 13.5, 40.5.....
 - 42, 38, 34, 30,
 - 4, 9, 16, 25,
- A geometric sequence has an initial value of 18 and a common ratio of $\frac{1}{2}$. Write a function to represent this sequence. Graph the function.
- Write and graph the function that represents the sequence in the table.

x	1	2	3	4
$f(x)$	8	16	32	64