Math 1 **Geometric Sequences**

SWBAT write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

take note

Key Concept Geometric Sequence

A geometric sequence with a *starting value* a and a *common ratio* r is a sequence of the form

$a, ar, ar^2, ar^3, \ldots$

A *recursive definition* for the sequence has two parts:

$a_1 = a$ Initial condition

$a_n = a_{n-1} \cdot r$, for $n \geq 2$ Recursive formula

An *explicit definition* for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

Every geometric sequence has a starting value and a common ratio. The starting value and common ratio define a unique geometric sequence.

Essential Understanding: In a geometric sequence, the ratio of any term to its preceding term is a constant value.

The recursive formula is useful for finding the next term in the sequence.

The explicit formula is more convenient when finding the nth term

Identifying an Geometric Sequence. The common ratio between every pair of consecutive terms must be the same.

Example 1: **Example 2:**

Example 3:

A 20 200 2,000 20,000 200,000, . . .

× 10 × 10 × 10 × 10

B 2 4 6 8 10, . . .

× 2 × 1.5 × 1.33 × 1.25

C 5 −5 5 −5 5, . . .

× −1 × −1 × −1 × −1

There is a common ratio, $r = 10$. So, the sequence is geometric.

There is no common ratio. So, the sequence is not geometric.

There is a common ratio, $r = -1$. So, the sequence is geometric.

Tell whether the sequence is geometric. If the sequence is not geometric, is it arithmetic?

a. 3, 6, 12, 24, 48,…. b. 3, 6, 9, 12, 15… c. 1/3, 1/9, 1/27, 1/81…. d. 4, 7, 11, 16, 22, …..

Find the recursive and explicit formulas for the sequence 7, 21, 63, 189,….

The starting value a_1 is 7. The common ratio r is $\frac{21}{7} = 3$.

$a_1 = a; a_n = a_{n-1} \cdot r$	Use the formula.	$a_n = a_1 \cdot r^{n-1}$
$a_1 = 7; a_n = a_{n-1} \cdot r$	Substitute the starting value for a_1.	$a_n = 7 \cdot r^{n-1}$
$a_1 = 7; a_n = a_{n-1} \cdot 3$	Substitute the common ratio for r.	$a_n = 7 \cdot 3^{n-1}$

The recursive formula is $a_1 = 7; a_n = a_{n-1} \cdot 3$.

The explicit formula is $a_n = 7 \cdot 3^{n-1}$.

Practice: Find the 8th term of each sequence

a. 14, 84, 504, 3024

b. 648, 324, 162, 81

Writing Geometric Sequences as Functions: A geometric sequence has an initial value of 6 and a common ratio of 2. Write a function to represent the sequence. Graph the function.

Practice: A geometric sequence has an initial value of 2 and a common ratio of 3. Write a function to represent the sequence. Graph the function.

Homework:

1. Determine if each sequence is a geometric sequence. If it is, find the common ratio and write the explicit formula. Solve for the 9th term.
   a. 5, 10, 20, 40, ……
   b. 20, 15, 10, 5, ….
   c. 3, -9, 27, -81 …..
2. Identify each sequence as arithmetic, geometric or neither.
   a. 1.5, 4.5, 13.5, 40.5……
   b. 42, 38, 34, 30, …..
   c. 4, 9, 16, 25, …..
3. A geometric sequence has an initial vallue of 18 and a common ratio of ½ . Write a function to represent this sequence. Graph the function.
4. Write and graph the function that represents the sequence in the table.

x	1	2	3	4
f(x)	8	16	32	64